VACUUM DECAY WITH BLACK HOLES

SOUTHAMPTON – 18/11/21

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Motivation: Seeded Tunneling

One of the mysteries at the core of the conflict between gravity and particle physics is the nature of the vacuum. Assumptions about the vacuum underly Hawking radiation and inflation. This research arose from re-examining methods behind calculations of first order phase transitions in the universe.

Usually, first order phase transitions are triggered by impurities, e.g. raindrops, diet coke and mentos(!), and of course – proton deay! But our description of QM tunneling is much less “messy”.

Much ado about Nothing!

The simplest way to define the vacuum is to say it is the state of “lowest possible” energy.

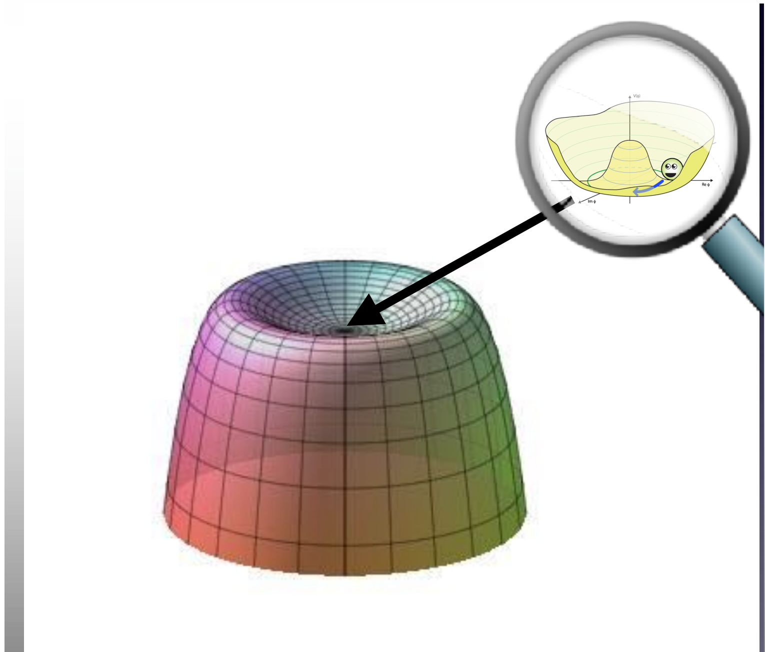
This presupposes that there is a lower bound, and that we are in it!

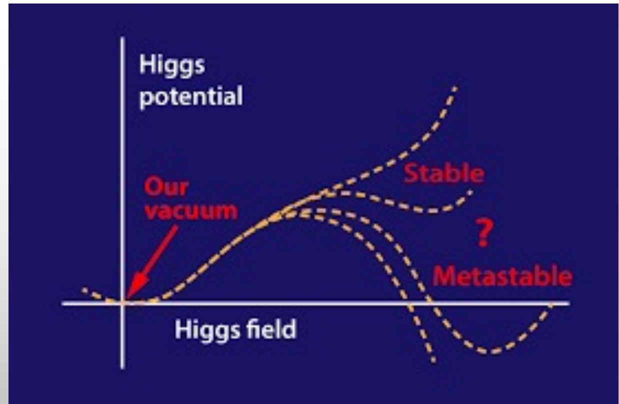
The Higgs field defines our vacuum – but there is more than meets the eye!

Higgs Vacuum?

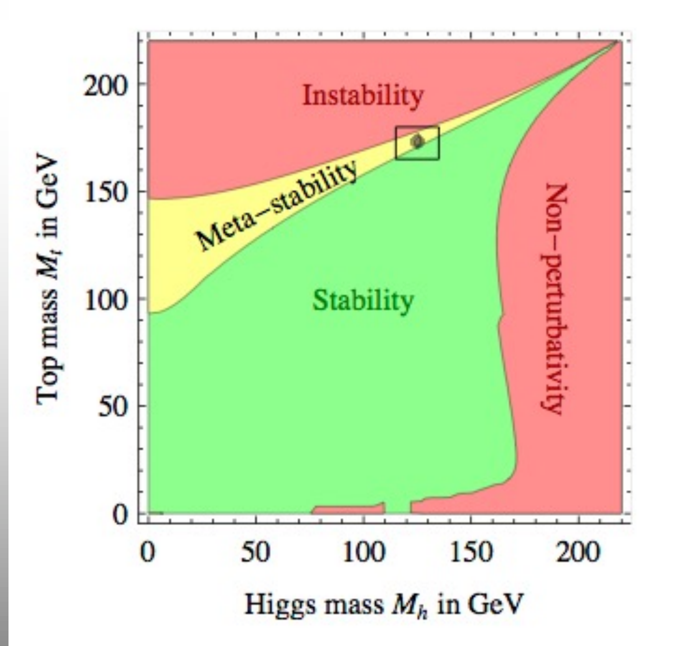
In the standard model, the Higgs field helps set the masses of the other particles. But the mechanism gets corrections according to the energy scale.

The calculation depends particularly on the highest mass top quark – that is also the least precisely known. Calculating the running of the Higgs coupling seemed to tell us we are in a sweet spot between stability and instability – metastability.





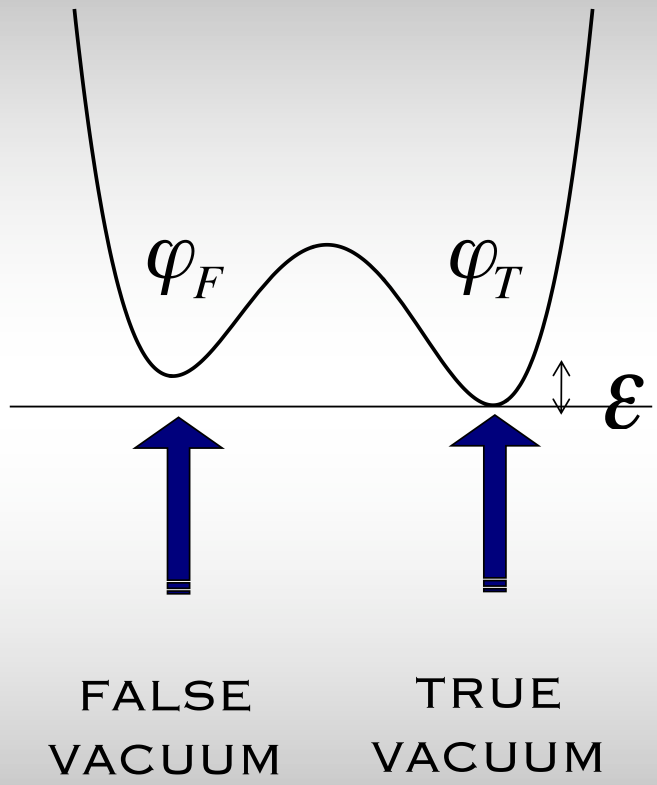
Though see 1904.05237 (CMS) & 1905.02302 (ATLAS).



Vacuum Decay

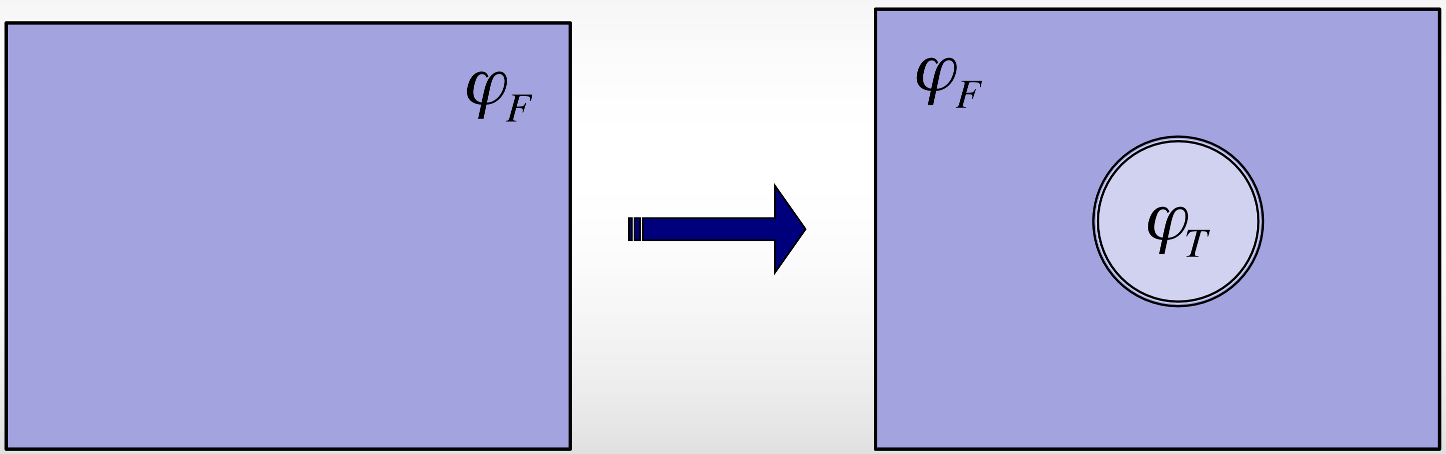
What this means is that if the Higgs takes a different (and large) value – the vacuum energy is lower.

This will give a first order phase transition, where we tunnel from one local energy minimum to another with lower overall energy.



QFTunnelling

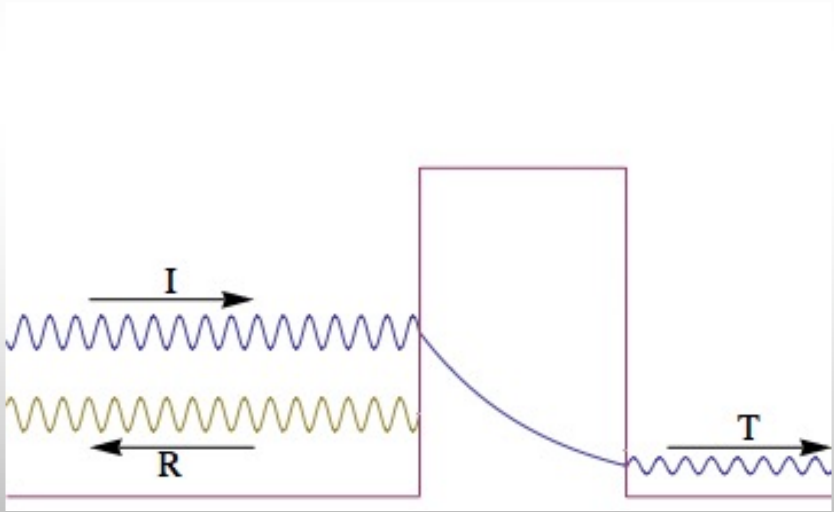
Developed by Coleman and others in the 1970's. It describes vacuum decay via a mathematical tool of analytically continuing to imaginary (Euclidean) time.



Coleman described this in field theory by the Euclidean solution of a bubble of true vacuum inside false vacuum separated by a "thin wall". We solve these Euclidean equations and compute the action.

Motivation

To motivate the calculation, step back to 1st year QM.
First meet tunneling in the Schrodinger equation. Standard 1+1 Schrodinger tunneling exactly soluble. Recall tunnelling probabilities exponentially suppressed.



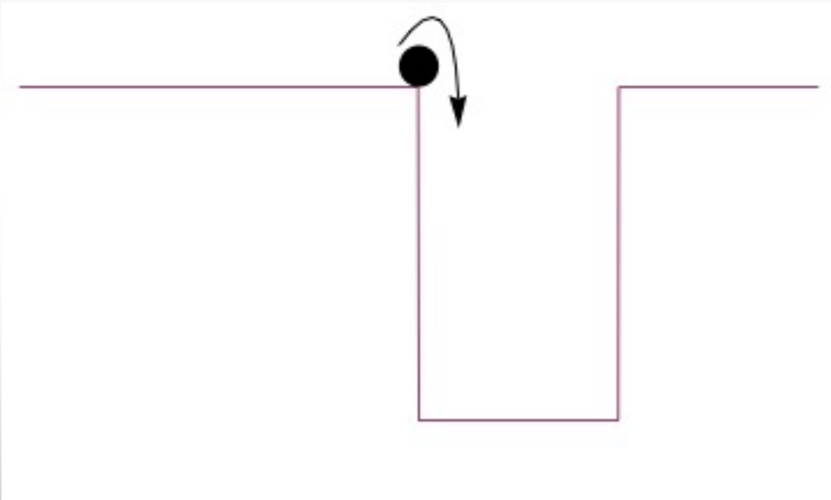
$$|T|^2 = \frac{1}{1 + \frac{V_0^2 \sinh^2 \Omega d}{4E(V_0 - E)}} \approx e^{-2\Omega d}$$

$$\Omega^2 = \frac{2m}{\hbar^2}(V_0 - E)$$

$$\Omega d = \frac{1}{\hbar}\int_0^d \sqrt{2m(V_0 - E)}dx$$

Euclidean Trick

A simple and intuitive way of extracting this leading order behaviour is to take "classical" motion in Euclidean time:



$$\boxed{t \to i\tau}$$

$$\frac{1}{2}\left(\frac{dx}{d\tau}\right)^2 = \Delta V$$

$$\int \sqrt{2\Delta V}\,dx = \int 2\Delta V\,d\tau = \int \left(\Delta V + \frac{1}{2}\left(\frac{dx}{d\tau}\right)^2\right) d\tau = \int L_E\,d\tau$$

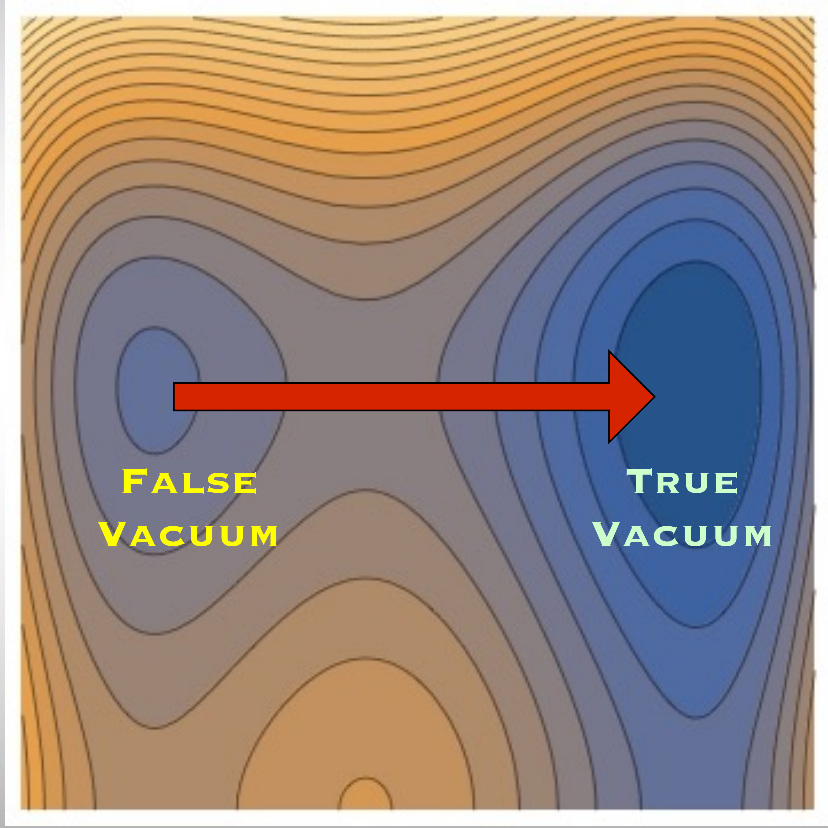
Euclidean 'Motion'

For more general potentials, this gives an intuitive visualisation of the tunneling amplitude calculation.
The particle rolls from the (now) unstable point to the "exit" and back again – a "bounce".

In QFT, we construct instantons as solutions to the Euclidean equations of motion. Can think of this by analogy to Schrodinger problem, or via functional Schrodinger approach.

Most Probable Escape Paths

This picture was generalised by Banks Bender and Wu to describe multi-dimensional tunnelling, that then motivates the field theory Euclidean approach.



Via quantum uncertainty, a bubble of true vacuum suddenly appears in the false vacuum – then expands

Goldilocks Bubble

But we can find the answer by and intuitive argument: If a bubble fluctuates into existence, we gain energy from moving to true vacuum, but the bubble wall costs energy.

Too small and the bubble has too much surface area – recollapses.

Too large and it is too expensive to form.

"Just Right" means the bubble will not recollapse, but is still "cheap enough" to form.

Goldilocks Bubble

Let us figure out what happens. If a bubble fluctuates into existence, we gain energy from moving to true vacuum, but the bubble wall costs energy:

$$\delta E = 2\pi^2 R^3 \sigma - \frac{\pi^2}{2} \varepsilon R^4$$

COST OF WALL

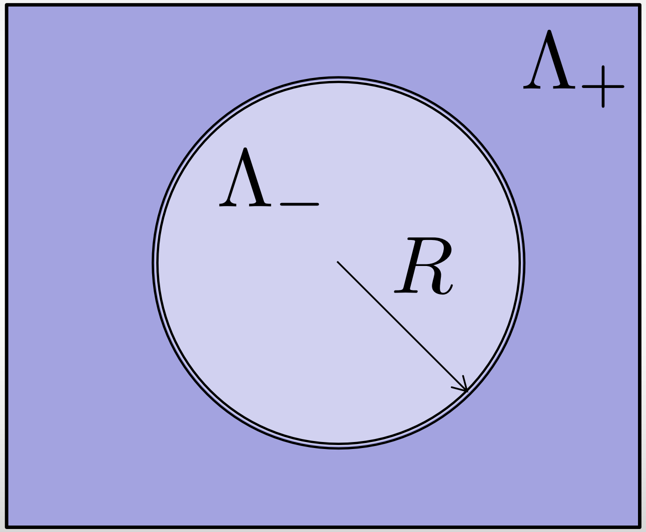
GAIN FROM VACUUM

Euclidean Action

This corresponds beautifully to the Euclidean calculation of the tunneling solution: "The Bounce"

ENERGY COST $\sigma \times 2\pi^2 R^3$

ENERGY GAIN $\varepsilon \times \pi^2 R^4/2$



Solution stationary wrt R,

$$\Rightarrow R = 3\sigma/\varepsilon$$

Coleman Bounce

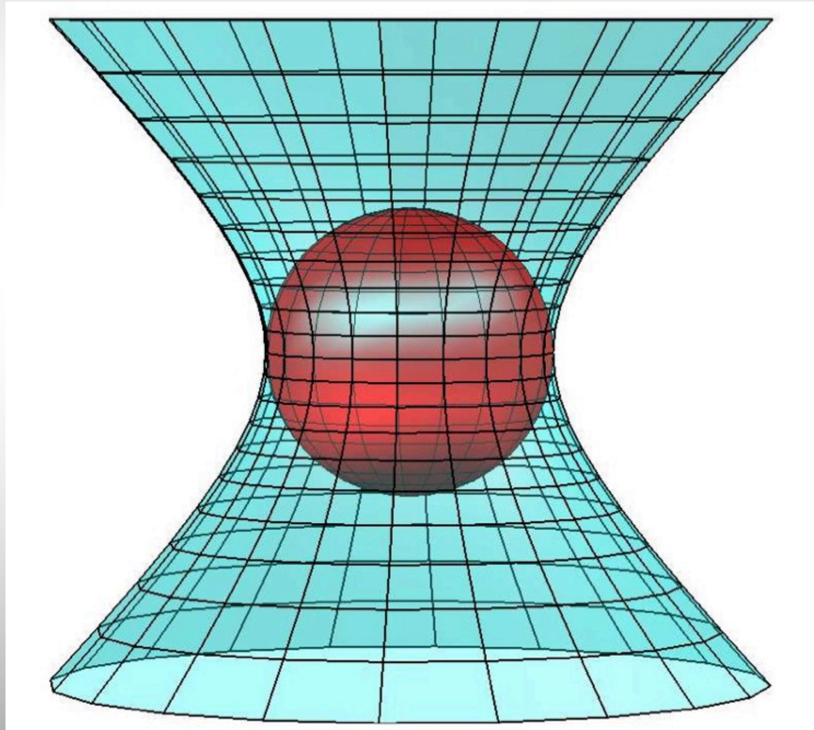
This gives us the bubble radius, and the amplitude for the decay – backed up by full field theory calculations.

$$\mathcal{B} = \frac{\pi^2 R^3}{2}\left(-\sigma + \varepsilon R\right) \sim \frac{27\pi^2}{2}\frac{\sigma^4}{\varepsilon^3}$$

This gives the leading order or saddle point approximation to the amplitude:

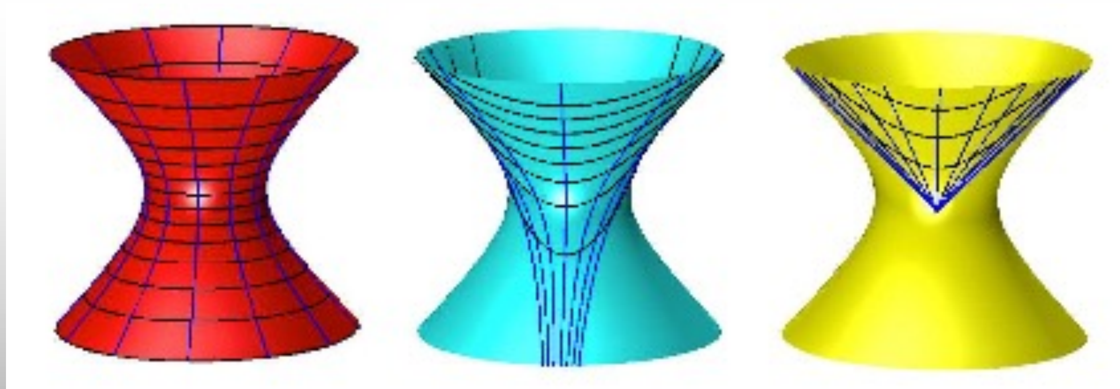
$$\mathcal{P} \sim e^{-\mathcal{B}/\hbar}$$

We then analytically continue back to real time, the spherical bubble becomes a Lorentzian hyperboloid – a bubble expanding in time.



Gravity and the Vacuum

This is not the full story! Vacuum energy gravitates – e.g. a positive cosmological constant gives us de Sitter spacetime – so we must add gravity to this picture



(Quantum) Gravity and the Vacuum

This is not the full story! Vacuum energy gravitates – e.g. a positive cosmological constant gives us de Sitter spacetime – so we must add gravity to this picture.

Although we do not have an uncontested theory of quantum gravity, we do have well-tested* methods on how to incorporate quantum effects in gravity below the Planck scale.

*For a theorist! Well-trodden might be more accurate!

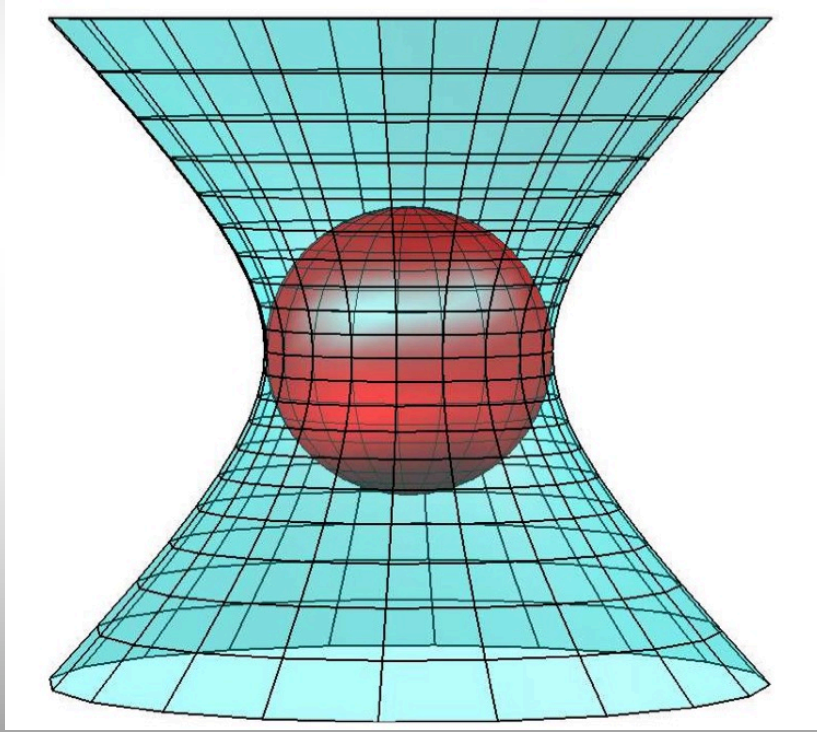
Gibbons-Hawking Euclidean Approach

Extend partition function description to include the Einstein-Hilbert action – at finite temperature we take finite periodicity of Euclidean time.

$$S = -\frac{M_p^2}{2}\int d^4x\sqrt{|g|}R + \int d^4x\mathcal{L}_{SM}$$

Fluctuations treated with caution, but saddle points unambiguous.

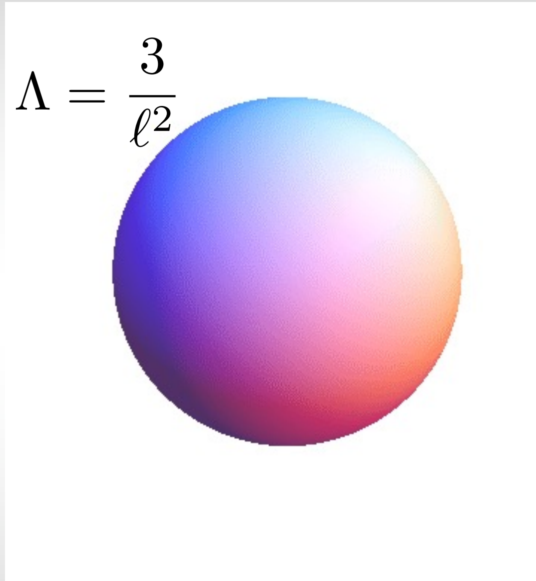
De Sitter spacetime has a Lorentzian (real time) and Euclidean (imaginary time) spacetime. The real time expanding universe looks like a hyperboloid and the Euclidean a sphere:

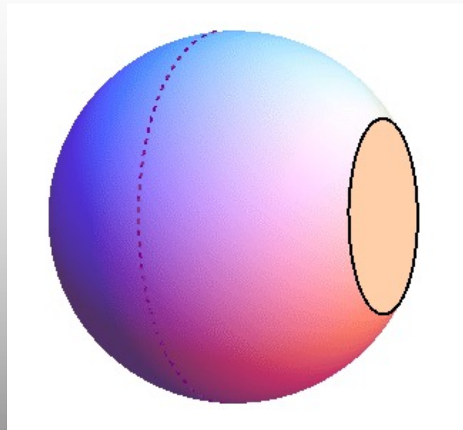


Our instanton must cut the sphere and replace it with flat space (true vacuum).

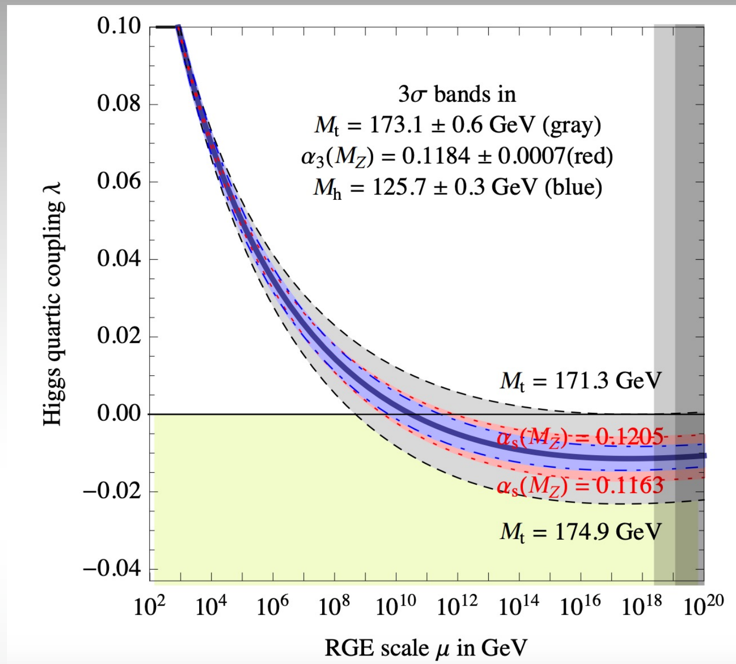
Coleman de Luccia (CDL)

Coleman and de Luccia showed how to add gravity by solving Euclidean Einstein equations. Euclidean de Sitter space is a sphere, of radius ℓ related to the cosmological constant. The true vacuum with zero cosmological constant, so must be flat. The instanton is a truncated sphere and can play same Goldilocks game (carefully!)





Coleman and de Luccia, PRD21 3305 (1980)



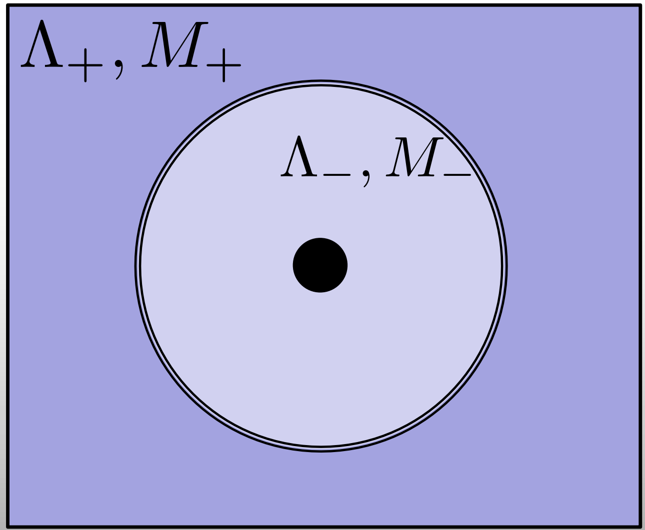
For the Higgs, this gives a half-life of many hundreds of billions of years.

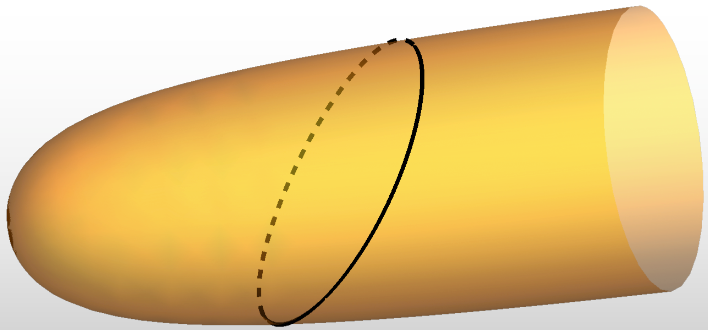
Question:

This picture is hugely idealized – the Coleman universe is empty and featureless – what if we throw in a little impurity?

Tweaking CDL

A black hole is an inhomogeneity, and also exactly soluble:





RG, Moss & Withers, 1401.0017

Key Steps:

- We can play the same "Goldilocks Bubble" game – the "just right" bubble will solve the Euclidean Einstein solutions.
- We then calculate the action of the bubble – much lower than CDL!
- Finally, we do a reality check on our method and compare to other physical black hole decay processes (evaporation!)
- (In reality we do this for SM bubbles which are very, very thick!)

Goldilocks Black Hole Bubbles

- The bubble with a black hole inside, can have a different mass term outside (seed).
- The solution in general depends on time, but for each seed mass there is a unique bubble with lowest action.
- For small seed masses this is time, but the bubble has no black hole inside it – no remnant black hole.

- For larger seed masses the bubble does not depend on Euclidean time, and has a remnant black hole.

This last case is the relevant one – the action is the difference in entropy (area) between the seed and remnant black holes!

Technical Aside:

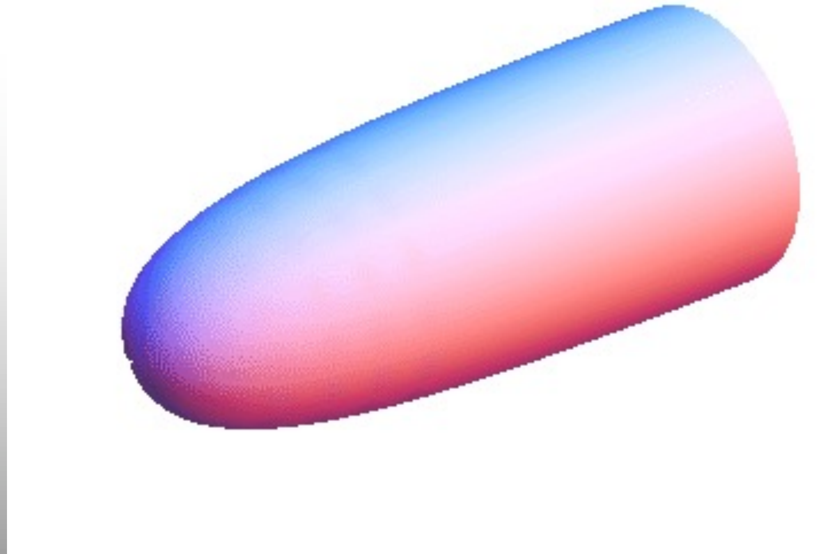
Euclidean Black Holes

In Euclidean Schwarzschild, to make the black hole horizon regular, we must have τ periodic. This "explains" black hole temperature, but also sets a specific value, $8\pi GM$.

$$ds^2 = \left(1 - \frac{2GM}{r}\right) d\tau^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega_{II}^2$$

$$\sim \rho^2 d\left(\frac{\tau}{4GM}\right)^2 + d\rho^2 + (2GM)^2 d\Omega_{II}^2$$

$$\rho^2 = 8GM(r - 2GM) \qquad \tau \sim \tau + 8\pi GM$$

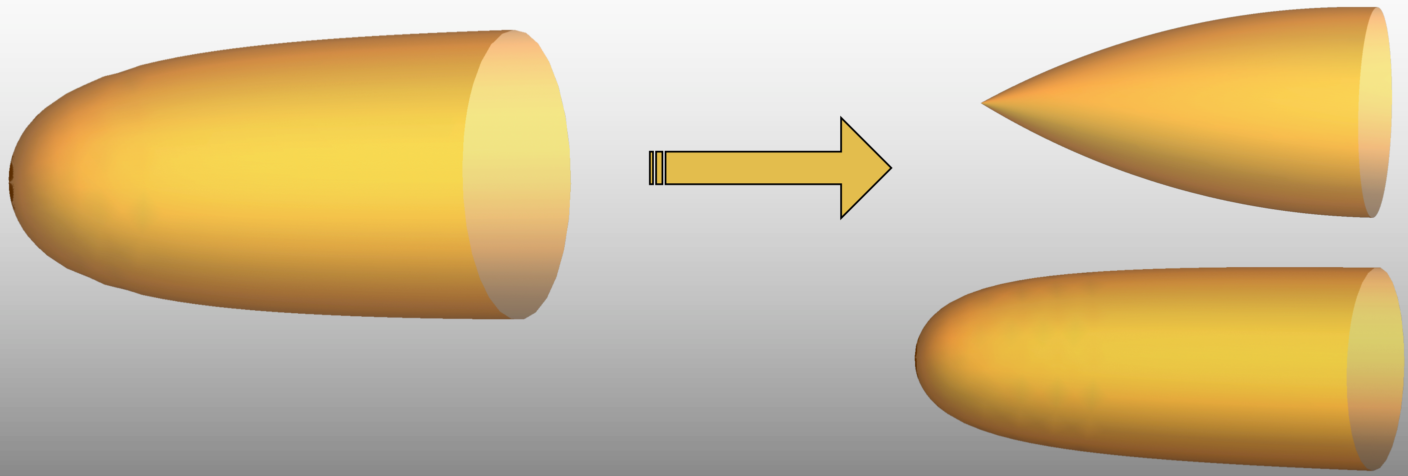


Technical Aside:

Conical Deficits

For different seed and remnant masses the periodicity is different – we need to deal with conical deficits. This technicality is **crucial** to the calculation, and gives a much lower instanton action.

To subtract off the false vacuum background, we must shrink the time circles to fit



Black Hole Bounces

Can think of the balance of action in Goldilocks argument changing because of periodic time:

$$B \sim \sigma \times 4\pi R^2 L - \varepsilon \times \frac{4}{3}\pi R^3 L$$

$$R \sim 2\sigma/\varepsilon$$

$$B \sim \frac{\sigma^3}{\varepsilon^2} L$$

The result is that the action is the difference in entropy of the seed and remnant black hole masses:

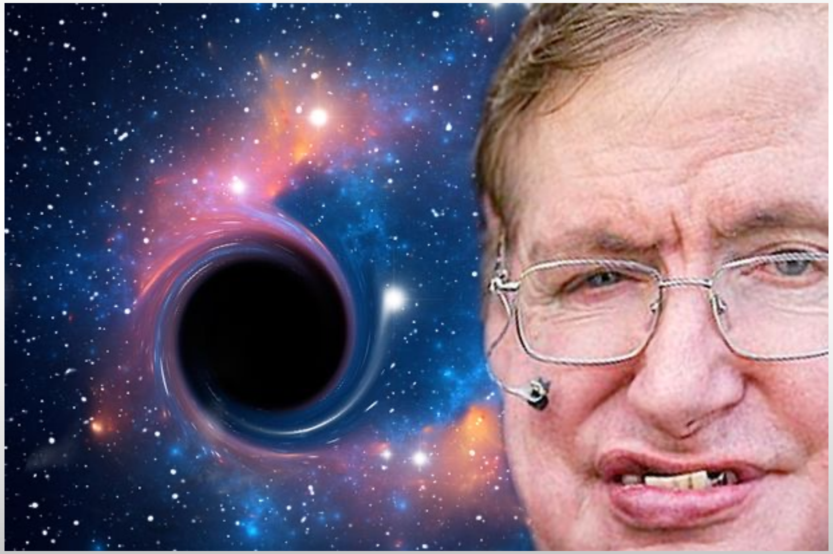
$$B \sim \mathcal{A}_+ - \mathcal{A}_-$$

Seeded tunneling is much more likely than CDL!

The fate of the black hole?

Vacuum decay is not all that can happen! Hawking tells us that black holes are black bodies, and radiate:

$$T_H = \frac{\hbar c^3}{8\pi G M k_B}$$



So we must compare evaporation rate to tunneling half-life.

Tunneling v Evaporation

Although we have computed bubble actions in full, we can estimate the dependence of the action on mass using input from our solutions which show that the seed and remnant masses are very close:

$$\begin{aligned}\mathcal{B} &= \pi(r_s^2 - r_r^2) \\ &\sim 4\pi(M_s + M_r)(M_s - M_r) \\ &\sim 8\pi M_s \delta M\end{aligned} \qquad \Rightarrow \Gamma_D \propto e^{-8\pi M_s \delta M}$$

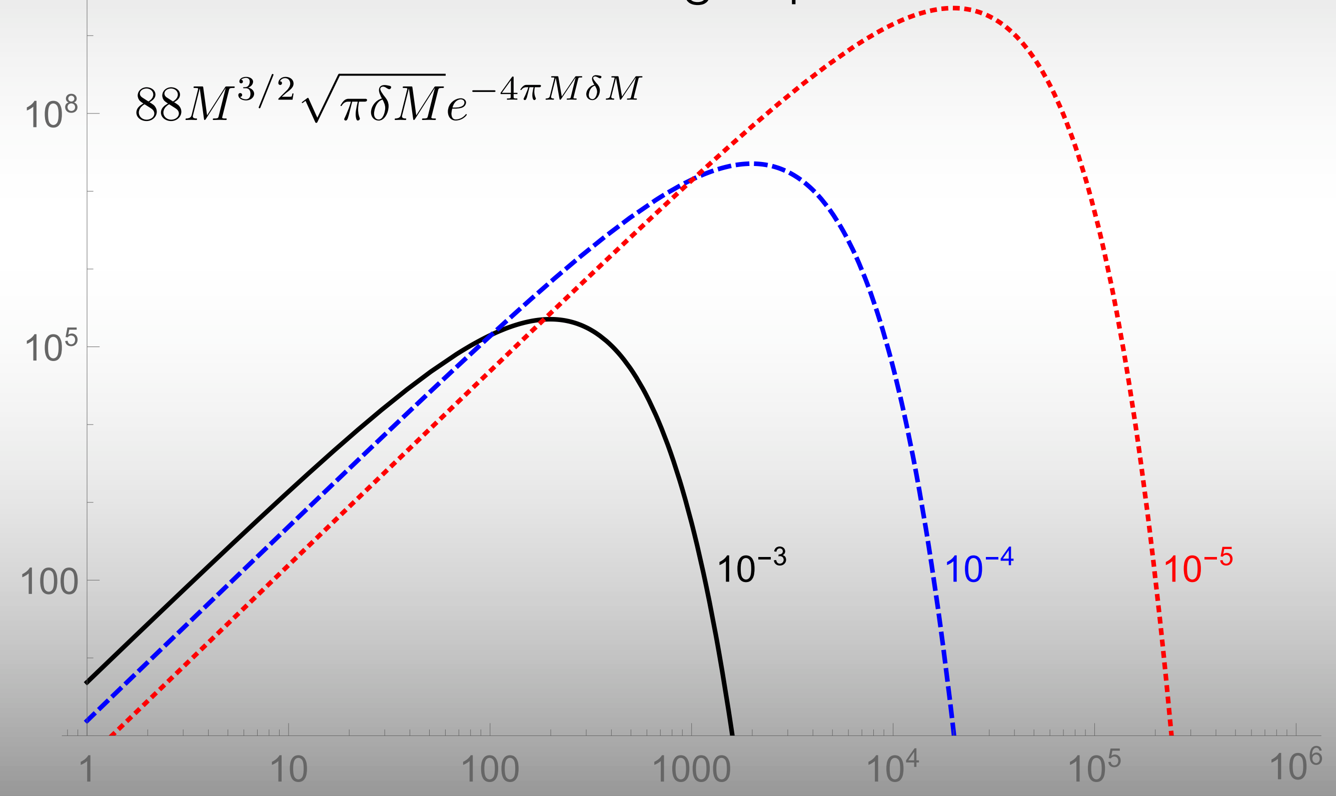
So our decay rate depends on an exponential of M_s, whereas evaporation depends on an inverse power of M – tunneling becomes important for smaller M

$$\Gamma_H \approx 3.6 \times 10^{-4} M^{-3}$$

Page, PRD-76

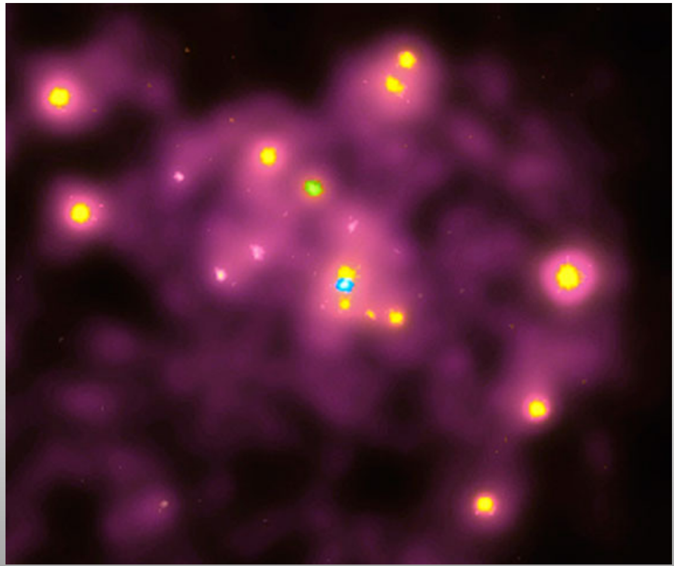
Tunneling v Evaporation

Taking this branching ratio estimate (in Planck units) shows how the dominance of tunnelling depends on δM and M:



Primordial Black Holes

In other words – only a primordial black hole will catalyse vacuum decay! Those primordial black holes that start out with small enough mass to evaporate will eventually hit these curves! And since the evaporation rate is super-fast, this decay will be essentially instantaneous.

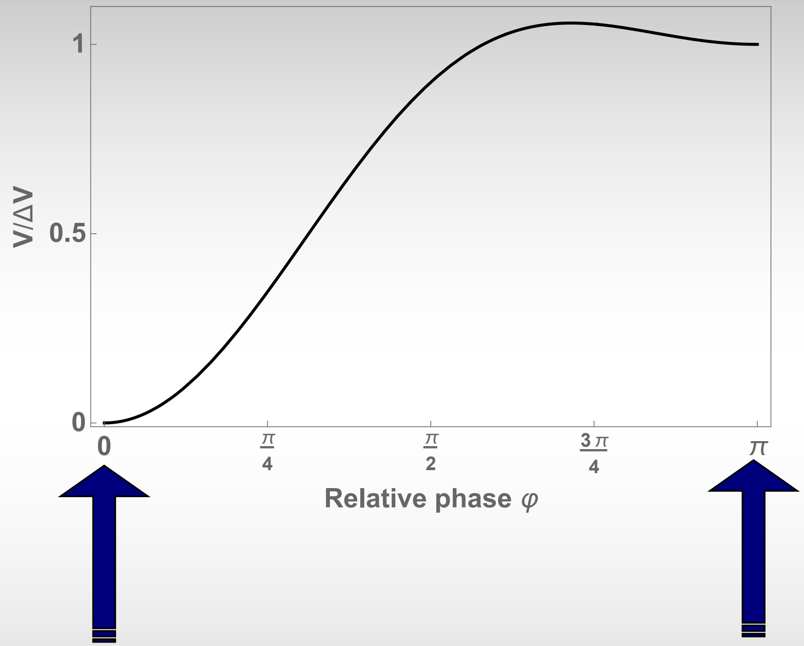


Testing Vacuum Decay

- The Euclidean method is a tool – but how much does it capture of the real process? Should we be trying other techniques?
- QM tunnelling well tested, but QFT tunnelling is another matter.
- Can we construct analog system that mimics a relativistic (or not!) metastable vacuum?
- And can we test seeded tunnelling?

Analog Counterpart?

Fialko et al proposed a table-top analog of false vacuum decay using a Bose gas in an optical trap with 2 different spin states coupled by a microwave field. Modulating the amplitude of this field stabilises a new false vacuum state* allowing vacuum decay to be potentially observed.

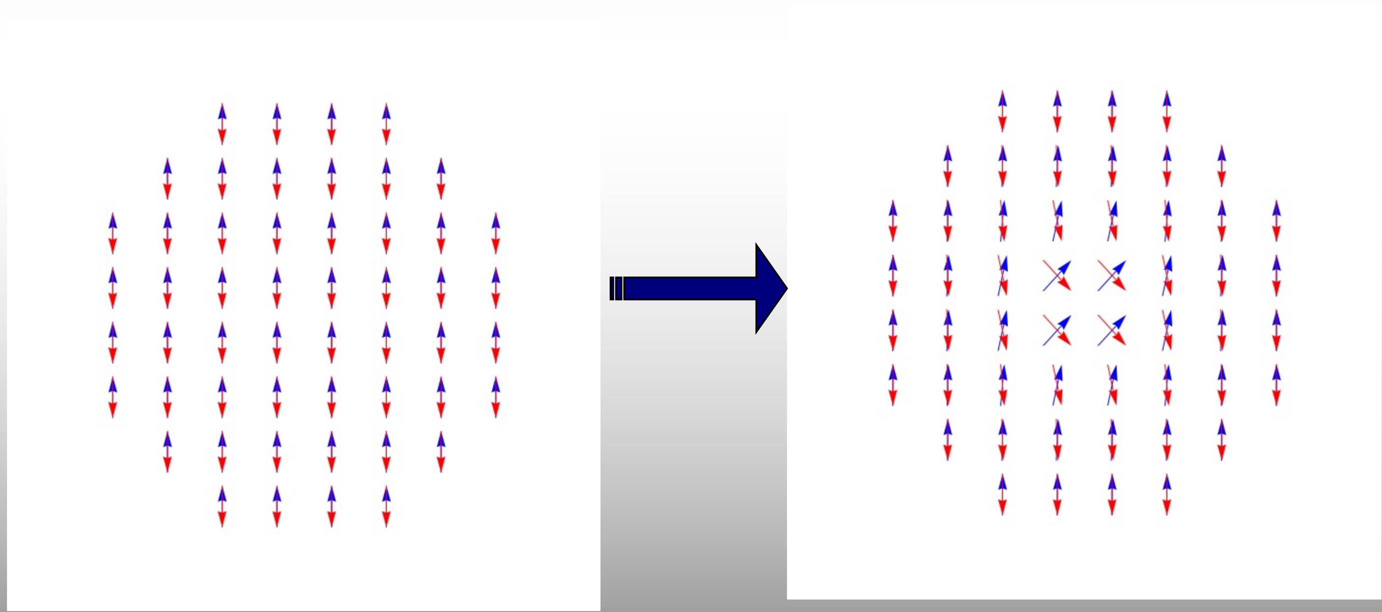


$$\Psi_1 = \Psi_2 \qquad \Psi_1 = -\Psi_2$$

Fialko, Opanchuk, Sidorov, Drummond, 1408.1163, 1607.01460
**Braden, Johnson, Peiris, Weinfurtner, 1712.02356*

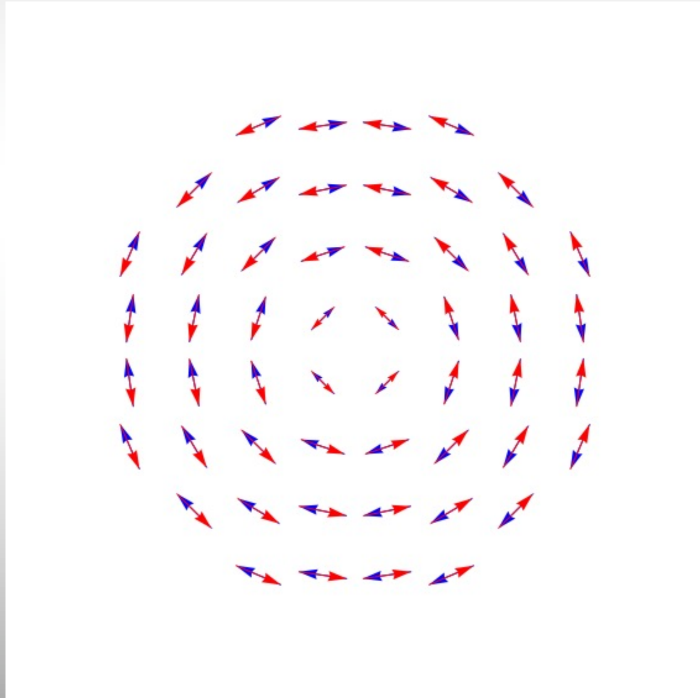
Coleman Decay

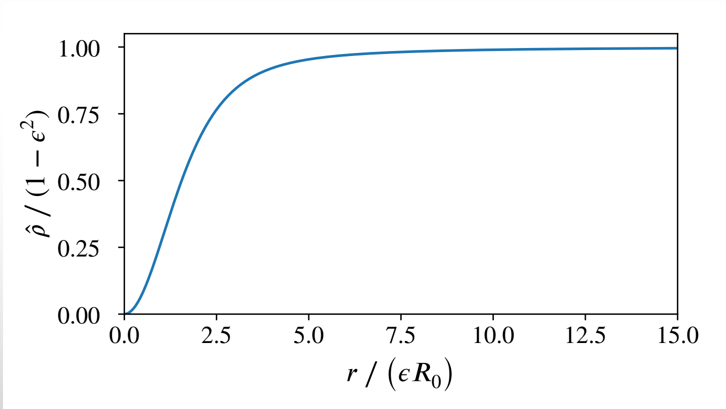
The false vacuum is uniform with constant density, the bubble interpolates to aligned wavefunctions at its center.



The Seed

The vortex has n=1 (or more) and exactly corresponds to a global vortex density field



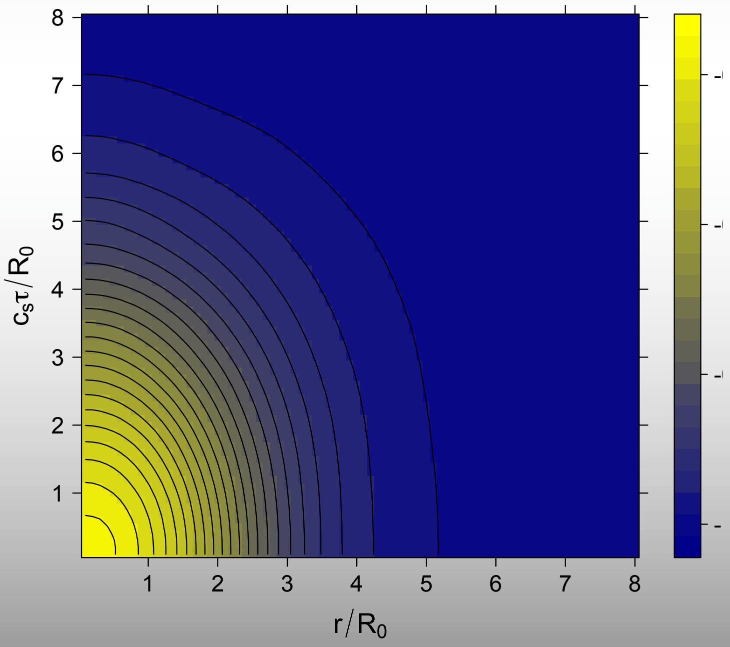


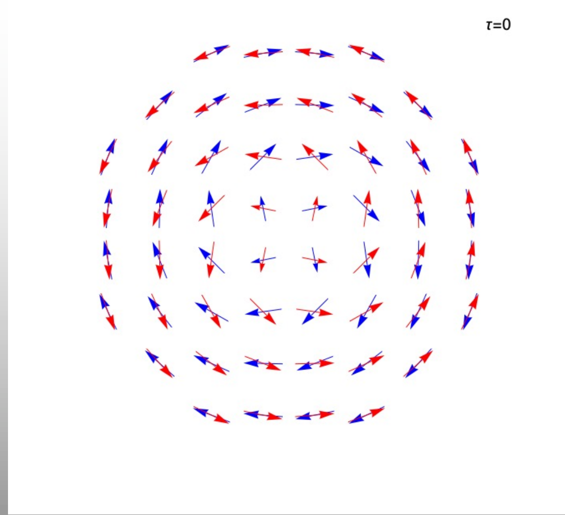
Seeded Bubbles

The bubble once again interpolates to (nearly) aligned phases near the core, with imaginary variation in density

$$\Psi_i = \rho^{1/2}\left(1 \pm \frac{\epsilon}{2}\sigma\right) e^{\pm i\varphi/2 + in\theta}$$

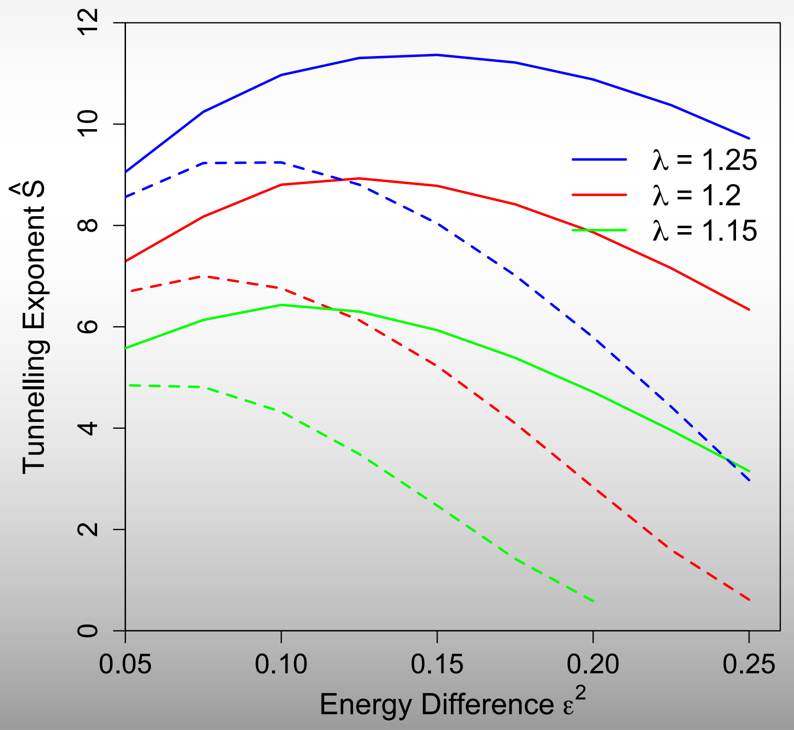
$$\bar{\Psi}_i = \rho^{1/2}\left(1 \pm \frac{\epsilon}{2}\sigma\right) e^{\mp i\varphi/2 + in\theta}$$





Seeded Amplitudes

Calculating the action of the instanton shows that it decreases for seeded tunneling:

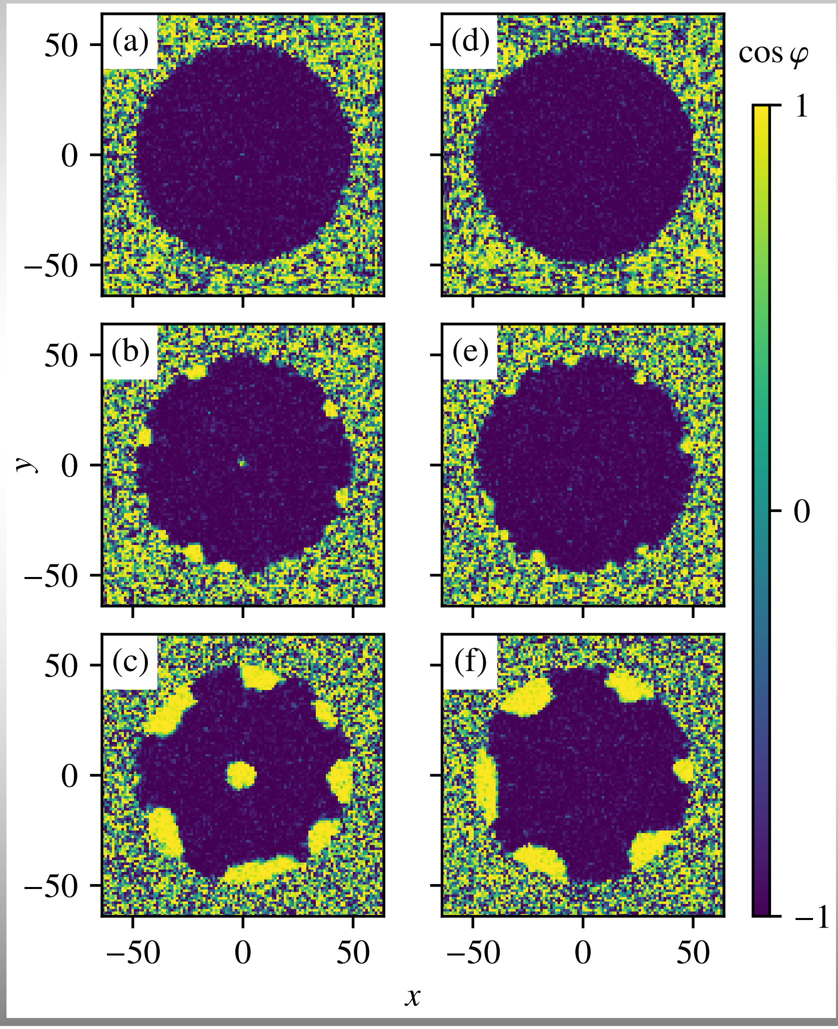


Truncated Wigner Method

An alternate method is to use the Gross Pitaevskii eqn to evolve the system, using zero temperature initial noise to populate the states of the system.

$$\Psi_i = \Psi_{iFV} + f(r)\sum_{\mathbf{k}} \beta_{i\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}$$

Results show the seed also instigates decay.



The team

"Growth comes through analogy; through seeing how things connect, rather than only seeing how they might be different." Albert Einstein

QSimFP

- St Andrews
- Newcastle
- KCL
- Nottingham
- Cambridge
- UCL
- RHUL

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Science & Technology Facilities Council

EPSRC Engineering and Physical Sciences Research Council

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- Anthony Kent
- John Owers-Bradley
- Xavier Rojas
- Viktor Tsepelin

Quantum circuits
- Gregoire Ithier

Quantum optics
- Friedrich Koenig

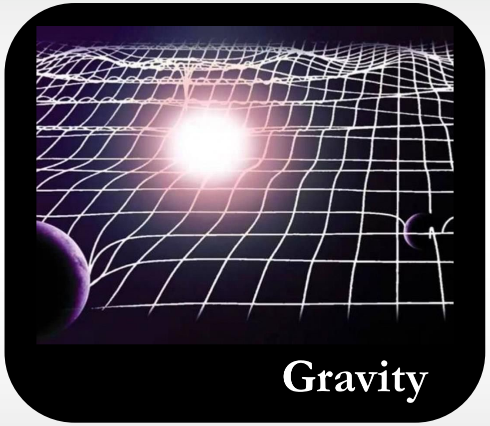
SW

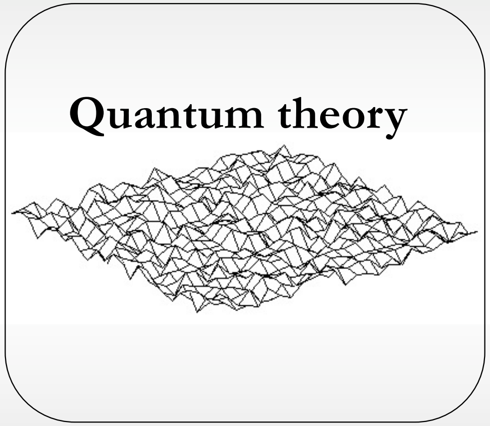
Motivation

- **Deepening our understanding of the dynamics of the early and late Universe** that arise in the inter-play between general relativity and quantum fields

- **Focus is on essential processes occurring in situations difficult/impossible to experiment with, and when conventional calculation techniques break down**
  → gravitational interactions are strong
  → quantum effects are important
  → length scales stretching beyond the observable Universe

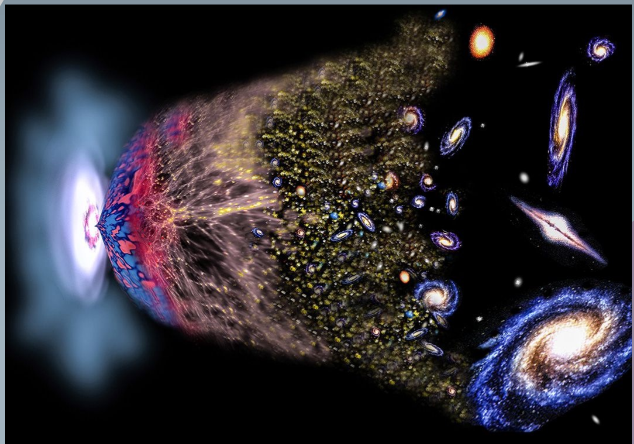
- **Introduce a 'cross-validation' between theory and experiment (here: validation of 'calculational toolboxes')**





SW

Objectives

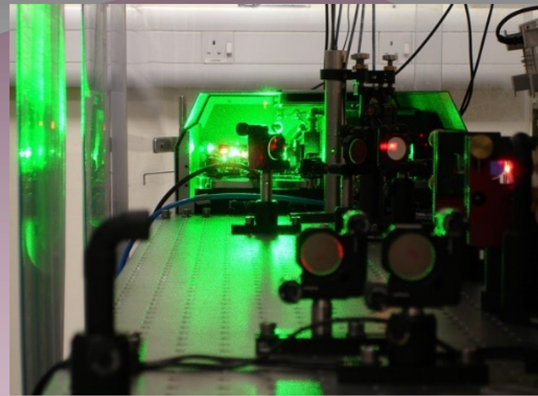


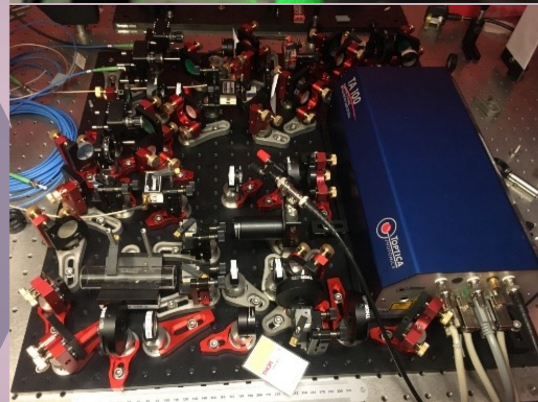
Quantum Vacuum:

- False Vacuum Decay
- Observer dependence

<u>Quantum Black Hole:</u>

- <u>Black hole ring-down</u>

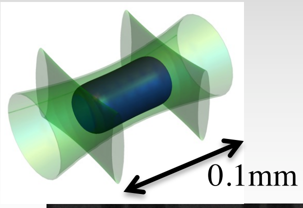


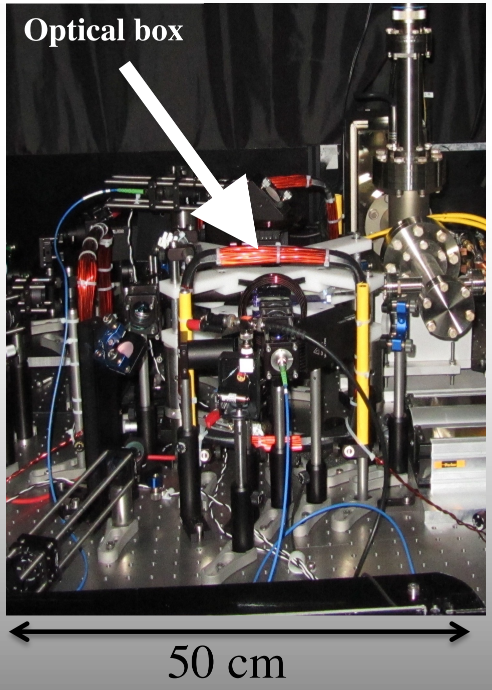


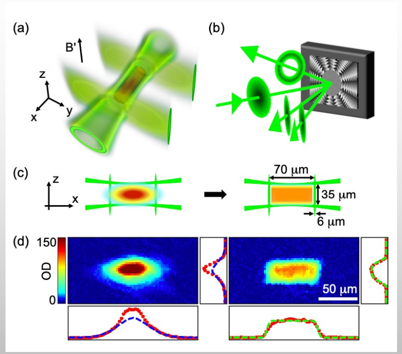
SW

FVD Workpackage

The false vacuum decay workpackage aims to test the process of relativistic vacuum decay via cold atoms in trap* whose effective theory is a relativistic vacuum.







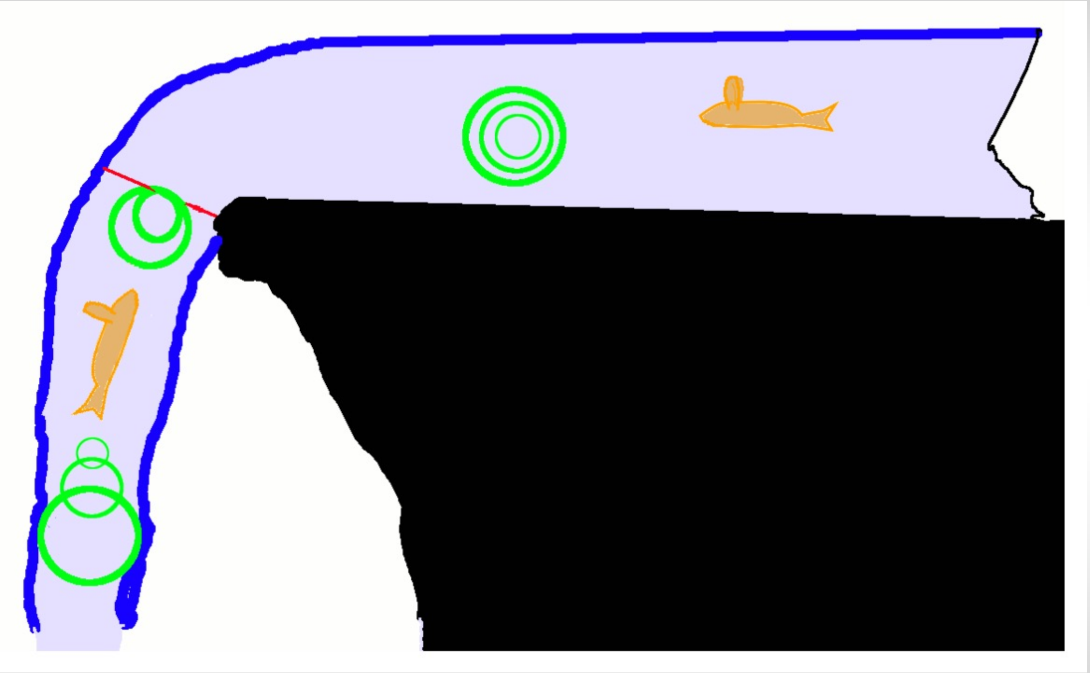
Zoran Hadzibabic, Science 347 (2015)
Zoran Hadzibabic, Nature 563 (2018)
Zoran Hadzibabic, Science 366 (2019)

* *A. L. Gaunt et al., Phys. Rev. Lett. 110, 200406 (2013).*

Analog "Black" Hole

- Key feature of a black hole is the causal boundary (event horizon) – the notion that information, or signals, cannot escape from a region.

- Unruh imagined a waterfall as a "dumb hole", if water flowing rapidly enough, sound from the unfortunate fish will not propagate back to the river.



Fluid Hole

More generally, the behaviour of long wavelength modes can often be described by an equation of the form (and usually the challenge is in understanding why!)

$$(\partial_t + \mathbf{v} \cdot \nabla)^2 \phi - c^2 \nabla^2 \phi = 0$$

We can rewrite this in tensor form:

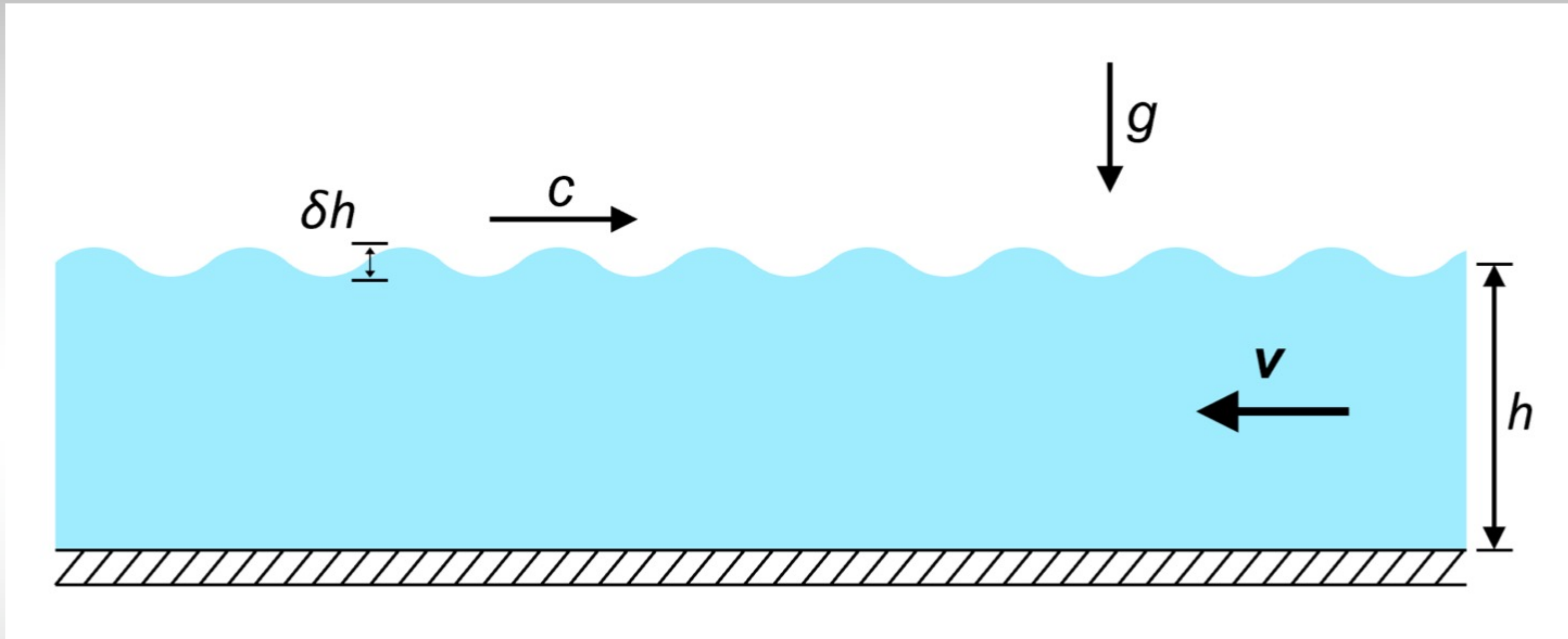
$$\frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \phi \right)$$

Where

$$g_{\mu\nu} = \begin{pmatrix} -(c^2 - v^2) & -v \\ -v & \mathbb{I} \end{pmatrix}$$

But this equation is super-familiar!
KLEIN-GORDON

Example: Surface Gravity Waves

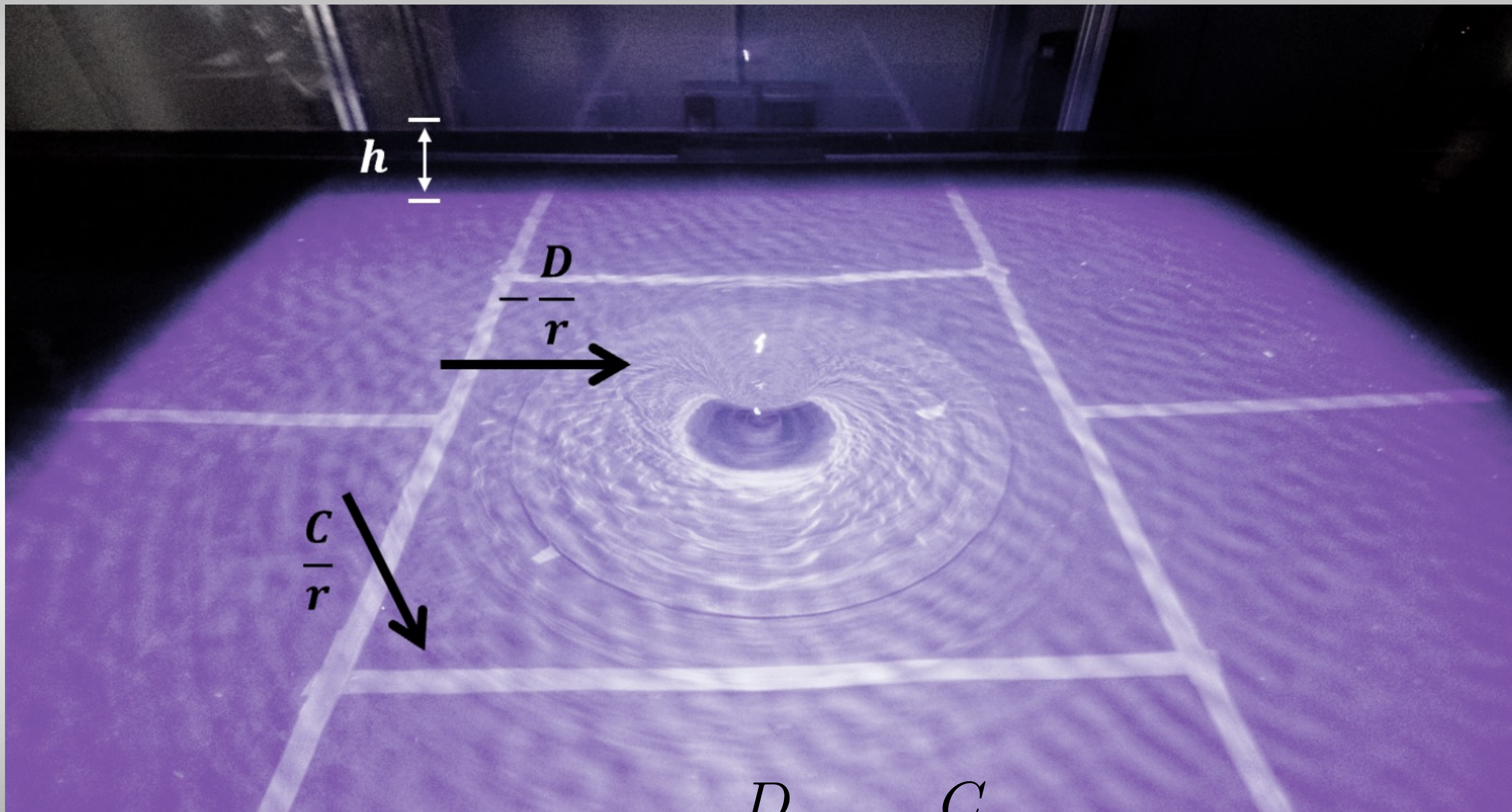


Equations of motion: $$(\partial_t + \mathbf{v}\cdot\nabla)^2\,\phi - ig\nabla\tanh\left(-ih\nabla\right)\phi = 0\,, \qquad \delta h = -\frac{1}{g}\left(\partial_t + \mathbf{v}\cdot\nabla\right)\phi$$

Long wavelength: $$(\partial_t + \mathbf{v}\cdot\nabla)^2\,\phi - c^2\nabla^2\phi = 0\,, \qquad c^2 = gh$$

SP

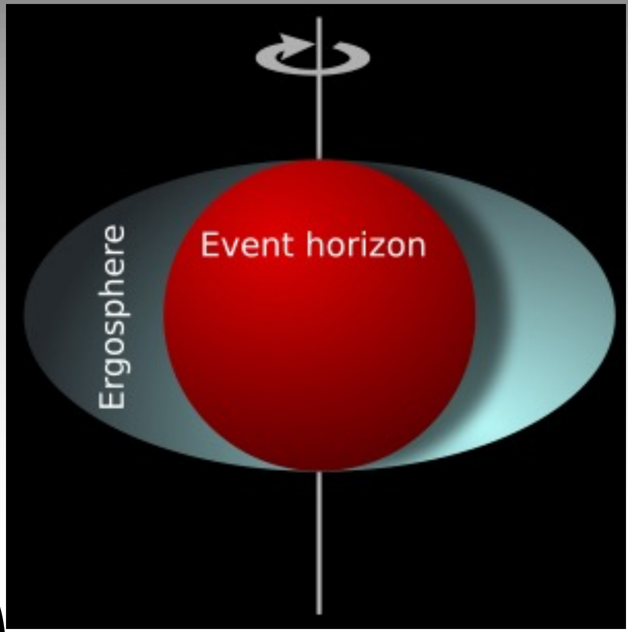
Experiment: Draining Bath Tub



Velocity profile, h roughly constant: $$\mathbf{v} = -\frac{D}{r}\mathbf{e}_r + \frac{C}{r}\mathbf{e}_\theta$$

SP

ANALOGY : KERR BLACK HOLE



Distinctive feature is the *ERGOSPHERE*, within which no observer can remain at rest relative to infinity. Its boundary is found where $g_{tt} = 0$.

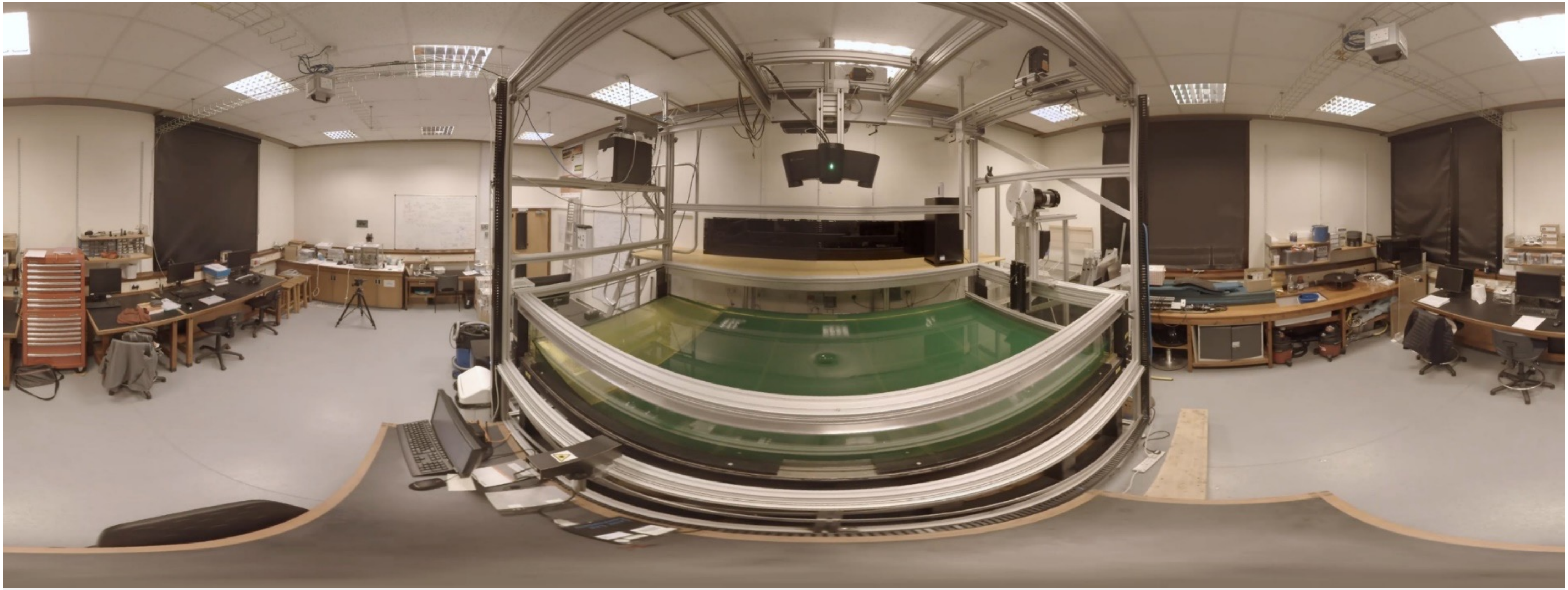
Here:

$$\mathbf{v} = -\frac{D}{r}\mathbf{e}_r + \frac{C}{r}\mathbf{e}_\theta \quad \Longleftrightarrow \quad g_{\mu\nu} = \begin{pmatrix} -\left(C^2 - \frac{C^2+D^2}{r^2}\right) & \frac{D}{r} & -C \\ \frac{D}{r} & 1 & 0 \\ -C & 0 & r^2 \end{pmatrix}$$

The velocity field gives both a horizon, as well as an ergoregion

$$r_s = D/c\,, \qquad r_e = \sqrt{C^2 + D^2}/c$$

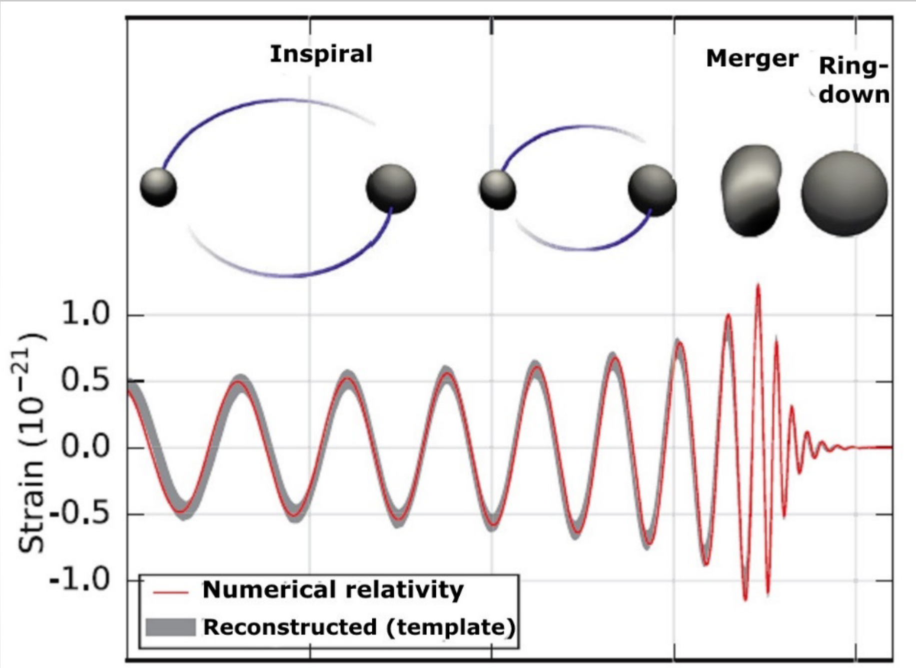
SP

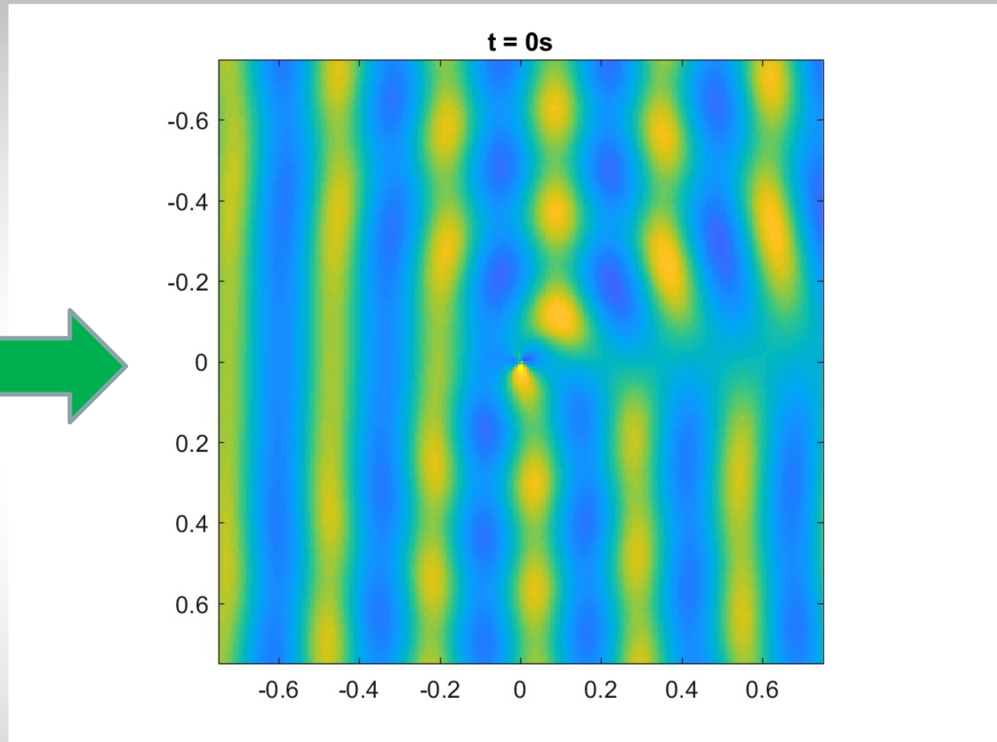


Building a Black Hole – Silke Weinfurtner

A sinkhole vortex provides a direct parallel to a rotating black hole

This wave propagation mirrors the ringdown of an excited black hole

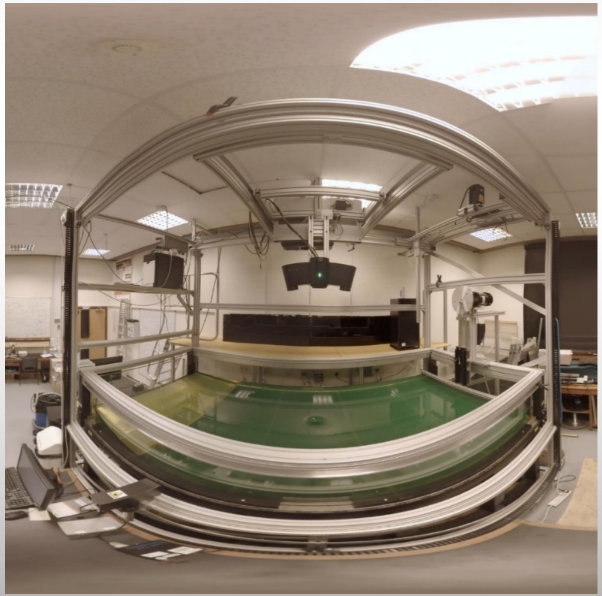




SW

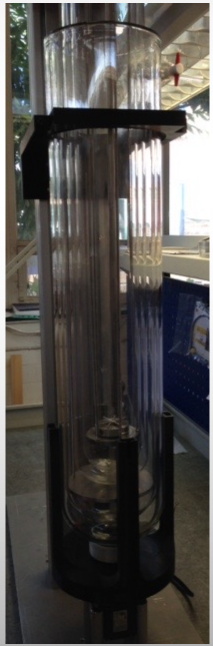
Experiments will have quantised angular momentum

Classical angular momentum
Classical surface waves



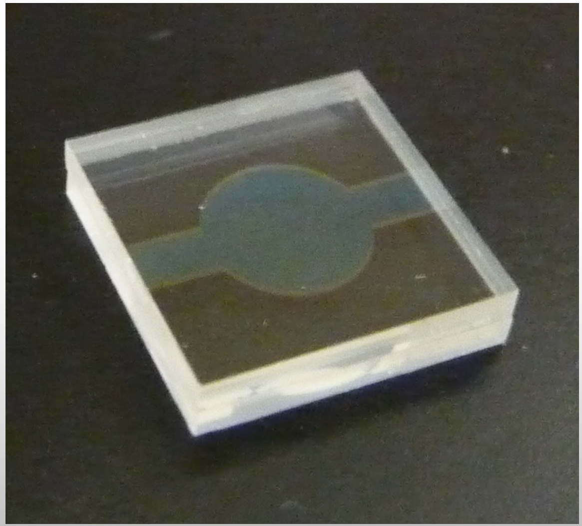
Classical spacetime
Classical relativistic fields

Quantised angular momentum
Classical relativistic ripplons



Quantum spacetime
Classical relativistic fields

Quantised angular momentum
Quantum relativistic ripplons



Quantum spacetime
Quantum relativistic fields

SW

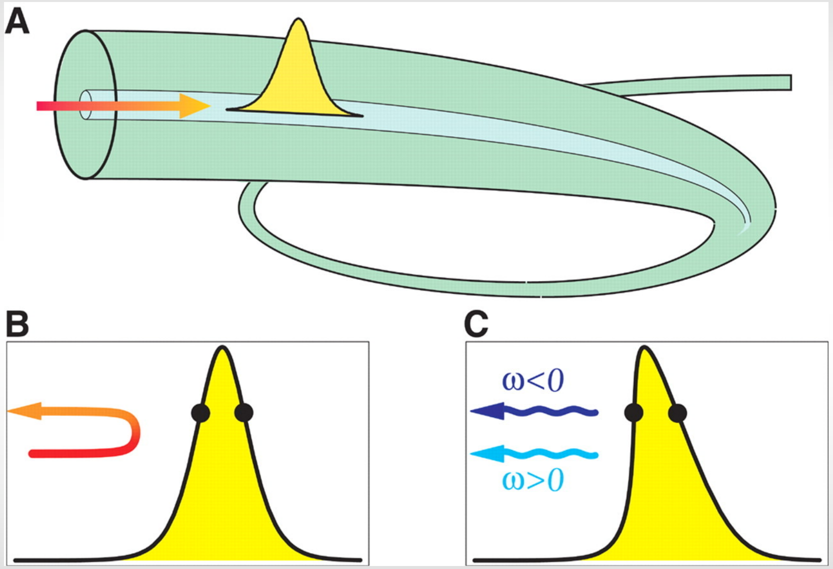
Experimental "Quantum" Black Hole (Nottingham)

Set up an interferometric detection method for fluid interfaces at room temperature.

Constructed a macro-scale cryogenic platform to characterise stationary draining superfluid vortex flows.

Currently identifying and minimizing heat-leaks and expect to reach temperatures between 1.5-2K by Feb 2022. They anticipate a ready-to-go experimental Quantum Black Hole Simulator at Nottingham starting Jun 2022.

Fibre Optic "Black" Holes



- **Black hole / white hole**:
  Light pulse in cable alters local refractive index, slowing down light probes. Demonstrates classical idea of black hole on forefront of pulse, white hole on trailing end.

- **Detection of ring-down modes**:
  Soliton oscillations and radiation are to be observed in the laboratory and put into a quantitative framework.

- **Measurement of quantum correlation**
  Detection of the spectral quantum correlations within the light emerging from the oscillating effective spacetime.

Frieder Konig, St Andrews (E) & Theo Torres, RG (T)

SW

Summary

- Vacuum decay is an example of quantum effects in action with gravity – we have good tools, but they are idealised.
- Tunneling amplitudes significantly enhanced in the presence of a black hole – bubble forms around black hole and can remove it altogether. Important if Higgs vacuum metastable.
- But there is plenty of new physics potential and the analyses are moving away from metastability.
- We plan to test this (and more) in the lab!